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Conceptualization of Approaches and Thought Processes Emerging in Validating of Model in Mathematical Modeling in Technology Aided Environment*

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Abstract

The aim of the present study is to conceptualize the approaches displayed for validation of model and thought processes provided in mathematical modeling process performed in technology-aided learning environment. The participants of this grounded theory study were nineteen secondary school mathematics student teachers. The data gathered from the video recordings received while participants were solving the given problems, the written responses related to the solution of the problems, the GeoGebra solution files and the observation notes taken by the researchers in the process of problem solving. The constant comparative analysis based on open, axial, and selective coding methods were used for data analysis through the grounded theory. The five sub-steps were emerged from data analyzing related to the process of the validation of model. It was determined that these steps were covered by discussing the unexpected results in real-life situations, comparing the real life results with the estimation based on the experiences or measurements, the problem data, the situations given in video and pictures and the decision-making about the adequacy of the model. In this study, it was seen that the technology-aided environment enriched the cognitive processes in validating step. With the help of this research's results, it was constructed a detailed description related to the cognitive processes which can be displayed for validating of model in mathematical modeling. It was considered that the study would bring a different perspective to the researchers towards the process of mathematical modeling.

Key Words

GeoGebra, Grounded Theory, Mathematical Modeling, Secondary Student Teachers, Technology-Aided Mathematical Modeling.

Mathematics curriculum applied in today's' education system (National Council of Teachers of Mathematics [NCTM], 1979, 1989, 1998, 2000; Skolverket, 1997; Swedish Ministry Education, 1994) dynamic two concepts draw attention. One of them is use of technology and the other mathematical

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modeling. Since the late of 20th century in various countries importance of mathematical modeling has risen and mathematical modeling started to take part in curriculums of every stage of education (Blomhøj & Kjeldsen, 2006; Niss, 1989). With the new perspective mathematical modeling in schools has started to be seen as the base of the models which students have created in real life (Freudenthal, 1973; Stevens, 2000; Streefland, 1993 as cited in English, 2006). According to Lingefjärd (2006), there is a need for individuals who are get on with technology, have capability of problem solving and mathematical modeling in every aspect of life. Clayton (1999) states that; one of the main goals of mathematics education is to teach students how they can use their knowledge and technological devices effectively and being aware of the importance of mathematical modelling. Studies in literature shows a need for the teachers who can develop alternative ways in problem solving and then use and interpret them and also who believe in the necessity of creating mathematical concept with the help of technology (Baki, 2002; Lingefjard, 2000; NCTM, 2000; Schoenfeld, 1992). Studies on the process of mathematical modeling emphasize that the process is complex and includes circular mental process and also verification is a main stage in mathematical modelling process (Berry & Houston, 1995; Biccard & Wessels, 2011; Blomhøj & Jensen, 2006; Blum & Leiß, 2007; Borromeo-Ferri, 2006; Cheng, 2010; Galbraith & Stillman, 2006; Galbraith, Stillman, Brown, & Edwards, 2007; Mason, 1988; Siller & Greefrath, 2010; Stillman, Galbraith, Brown, & Edwards, 2007; Voskoglou, 2006). Given that students are often unsuccessful in the verification of the model, it is important to consider their difficulties in the process (Peter-Koop, 2004; Pugalee, 2004; Stillman & Galbraith, 1998). Because; it is thought that such details can give clues about cognitive structure and the skills needed to be used in the verification stage of modelling process. The findings presented in this study, which is the first detailed research that mathematical modelling is intertwined with GeoGebra, are selected from the research which mathematical modelling process is conceptualized with the help of theorizing in technology supported environment (Hıdıroğlu, 2012).

Mathematical modelling, in general, is a dynamic method which enable to see the relationship between the problems in every area of life and the nature to be able to put forward, classify and generalize the relations in mathematical terms; and making conclusions from them (Fox, 2006). Blum (2002), defends that mathematical modelling represent both transition from real life to mathematics and the whole process in that transition. Researchers' viewpoints towards the mathematical modelling can vary according to the researchers' main goals, approaches they have been affected and their field of application.

Different perspectives are presented in defining mathematical modeling process (Abrams, 2001; Berry & Houston, 1995; Borromeo-Ferri, 2007; Cheng, 2001; Kaiser, 2005). For instance, when viewpoints towards mathematical modeling are considered; we are come across six different modeling viewpoints such as "fair", "contextual", "educational", "social-critical", "epistemological" and "cognitive" (Kaiser, 2005). One of these viewpoints; cognitive modeling aims to analyse and understand the cognitive processes occurred during mathematical modeling process (Kaiser, 2005). In analysis of such processes, features and structures of problems which are subjects of modeling are considered as well. Because; structures of problems naturally affect classification of modeling. It is known that various classifications are done about mathematical modeling (Berry & Houston, 1995; Skovsmose, 1994; Treffers, 1987; Williams, 1989).

Aim of experimental modeling is with basing on the data obtained in an experimental process to reach the mathematical model that represent the situation best (Thomas, Weir, Hass, & Giordano, 2010). In theoretical modeling, in forming the mathematical model; the theory is included more than the data (Berry & Houston, 1995). In simulation modeling, one of the priorities is to search the ideal situation for a new design (Berry & Houston, 1995; Thomas et al., 2010). In dimensional-analysis modeling, physical quantities are taken as the basic components and different strategies are used to reveal the possible relations between them (Berry & Houston, 1995).

Important names in cognitive modeling such as Blum, Borromeo, Ferri and Leiß have carried out a project (COM²) which aims to search how students' mathematical thinking process are shaped by analysing the behaviours of students and teachers and the interaction between each other in modeling processes in secondary school's mathematics courses with a cognitive perspective (Blum & Ferri, 2009). Another important study in cognitive modeling is DISUM project which is carried out by Blum and study group that they have searched modeling cycles of 9th grade students during modeling activity and the cognitive factors in which they have difficulty (Leiß, Schukajlow Blum, Messner, & Pekrun, 2010). When it comes to technology supported environment, Galbraith and Stillman's

(2006) study which includes 6 main components and 5 main stages and 31 sub stages that explain these 5 main stages draws the attention as the most detailed study. Mathematical modeling process which was reformed with theorizing perspective by Hıdıroğlu (2012) includes 8 main components, 7 main stages and 47 sub stages (see Figure 1).

In mathematical modeling process, firstly the complex real world situation is tried to be understood. Foreknowledge about the given and the desired things in the problem is shown by simplifying in order to give meaning to the statement of the problem. In short, the complexion of the real world situation is ended by analyzing the problem. Later in the process, a general solution strategy is put forward by considering the necessary strategical factors (such as the variable, constant, etc.), mathematical concepts, technological tools etc. By making assumptions accordingly, systematical structure is established and a model of the real world problem situation is reached. Throughout the process, the ideal solution advances on the model representing the real world situation and the data is classified according to the mathematical symbols, knowledge and skills. Mathematization is performed by obtaining the necessary "sub-mathematical model (SMM)"s utilizing the technology properly. SMMs' graphical and algebraic representations are utilized to reach the "mathematical model (MM)" by means of the SMMs obtained by using technology. Meta-mathematization is performed by associating SMMs according to the variables necessary for the MM. Benefiting from the obtained MM, the mathematical solution which is necessary to reach what is desired in the problem is obtained. Mathematical results are also gained by performing the mathematical analysis of the real world situation. In mathematical analysis, two concepts as mathematical solution and mathematical result show up. The mathematical solution faces us as mathematical expressions which are obtained from the MM and respond to the desired situation. However, the mathematical results are sometimes used to reach the mathematical solution and sometimes enable a general view of the MM for different situations of the real world situation. For these mathematical solutions and results to make sense in the real world situation, they need adapting to the real world. Interpreting/evaluating is done by scrutinizing the relationship between the mathematical world and the real world, and the real world solution is obtained from the mathematical solution and the real world results are obtained from the mathematical results. In the modeling process, after gaining the real world solution, it is observed that the accuracy of the real world results obtained from the model is scrutinized by utilizing the daily life experiences, the

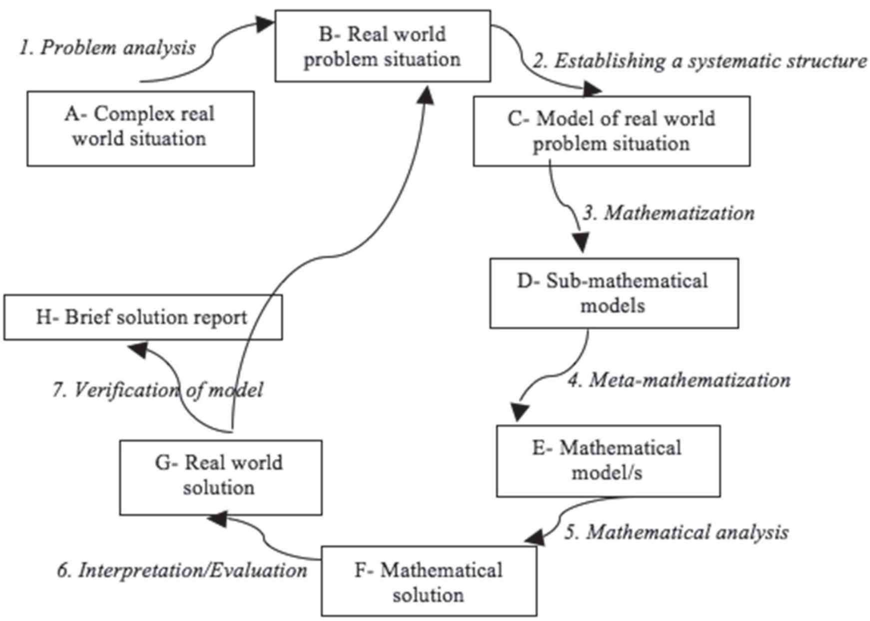


Figure 1.
Mathematical Modeling Process (Hıdıroğlu, 2012)

animations, videos/pictures given with the problems and the measurements that could be made during the solution. Besides, the accuracy of the MM is scrutinized too, by utilizing SMMs. The data which were obtained theoretically or experimentally about the real world problem is compared and the validity of the model is judged. This basic stage faces us as the basic stage containing the validation of the model by using real world results. In the stage validating the model, the validity of the model is questioned not only by using the real world solution but also by taking into consideration the real world solutions. If the validity of the model is satisfactory for the solver, the next component is the brief solution report. If the real world solutions of the model are thought to be unrealistic, then the problem is revised and the validity of the model is tried to be provided by turning back to the previous stages.

Method

This study is a grounded theory study which is one of a qualitative research designs. Grounded theory explains the some unknown results with the obtained data and the relations between these results (Glaser, 1978). Grounded theory perspective reveals the base of the behaviours, actions or descriptions in a situation and interconnected relationship between the models of different situations (Charmaz, 2006). While theorizing from the data; to compare the data always and enhance the categories, it is vital to abstract the data from descriptive level to a higher level of theoretic category and explain them (Glaser & Strauss, 2006). The reason behind the selection of grounded theory in this study is the desire to discuss not only the occurred events about mathematical modeling but also the features of these events and the relations and dimensions between them (Punch, 2005).

Instruments

In the study (1) three mathematical modeling problems designed by the researchers, (2) the solutions of the prospective teachers to these problems and the analysis of the video records containing their think aloud in the solution process, (3) written response papers containing problem solutions (4) GeoGebra solution files and (5) the observation notes taken in the solution process by the researchers were used as the data. The three modeling problems developed in the scope of the research were called "Height-Foot Length Problem", "Stadium Problem" and "Swing Problem".

Data Analysis

In the analysis of the data, the constant comparative analysis which is based on the grounded theory was used. In this process, the conceptualization of the process was enabled via open, axial and selective coding by taking in to consideration the approach of Strauss and Corbin (1990). In the beginning of the analysis, two researchers and a third expert researcher who is experienced in modeling independent of the research, coded the data independently of each other and at the end of this open coding, a code list consisting of 135 codes was composed. Over time open, axial and selective coding became the parts of an intertwined data analysis process. The relationships and the differences between the codes were found out by making comparisons between the data steadily and by asking the what-why-how questions. Thus, by comparing the different indicators in the data, more abstract concepts than experimental (empirical) data were reached. Questions like "What are the properties that make the process distinctive?", "Why is it important?", "How did it come out?", "What is the reason of its coming out?", "What are its effects?" were asked continuously throughout the process and the answers to explain the concepts, their properties and relations were sought. The coding process finished with selective coding which was made to determine the central categories in the higher-abstraction level after the axial coding which was made to connect the open codes to each other. The codes were compared in the framework of the interviews and discussions which the researchers made by gathering and the similarities and the differences within the basis of different codes were revealed. The codes which were given different names with the same meaning were collected under a code determined together. To emphasize the basic properties of the code better, sometimes the code names were changed, should the occasion arose new codes were added.

Results

According to the results obtained from the analysis of the data "Validation of Model" was shaped under five sub-stages (see Table 1). In stage of validation of model it was questioned that what real life results obtained from A/MM means for real life. Three component of stage was stated as real life solution, real life problem situation and brief solution report. In that point it was decided with the help of experience, data such as video/picture/table and measurements done.

Table 1.
7th Main Stage of Modeling Process

G-Real Life Solution		
	Sub-Stages	Basic Questions to be searched for answer
7. Validation of Model	**7a**-Examination of Sub-Mathematical Model's (AMM)/MM unexpected results in real-life situations.	In which situations A/MM was not sufficient? Does it create a problem for solution?
	7b-Comparison of A/MM's real life results with predictions or measurements based on experiment	Is it suitable to make predictions or measurement on the obtained output and model? To what degree do model and results of model meet with the predictions or measurement?
	7c-Comparison of A/MM's real life results with problem data.	Are model and results of model sufficient for real data given in problem and taught to be ideal?
	7d-Comparison of A/MM's real life results with the situations in video and pictures.	Do Model and results of model explain the situations in video and pictures?
	7e-Making decision on sufficiency of A/MM's relating to real life problem situation.	Do model and results of model give an ideal solution? What is the reason behind the conflict between results? What is the prominent thought, approach or mathematical concept in problem solving process and why is it important?
H-Brief Solution Report / B-Real Life Problem Situation		

7a- Examination of Sub-Mathematical Model's (AMM)/ Mathematical Model's (MM) Unexpected Results in Real-life Situations

After students had determined the real life results of model, they analyzed the situations that MM are insufficient in real life while searching for answers of the questions such as "In which situations MM was not sufficient?", "Does it create a problem for solution?" in detail. Such situations create an environment for analyzing of the effect of assumption put forward in solution on the validation of model and for advantage of probable change of assumptions.

7b- Comparison of A/MM's Real Life Results with Predictions or Measurements Based on Experiment

Students have interpreted real life results obtained from MM by comparing with predictions or measurements based on experiment to control the validation of model. In this sub stage they have analyzed the trueness of model in detail and tested the sufficiency of model by taking into account various situations.

7c- Comparison of AMM's Real Life Results with Problem Data

Students have questioned the trueness of model by comparing the data-given with the problem- with MM's real life results. Thus, they have analyzed whether real life solutions obtained with the help of AMM or MM provide an answer to real life problem situation.

7d- Comparisons of A/MM's Real Life Results with the Situations in Video and Pictures

The animation, video and pictures given with the problem have presented a visual picture of real life situation to the students. In this way, students have analyzed the validation of model in detail. In this sub stage it is seen that video parts or pictures which visualize the situation in point the best provide a suitable environment to control the validation of obtained real life results.

7e- Making Decision on Sufficiency of A/MM's relating to Real Life Problem Situation

In this sub stage students have decided on the sufficiency of MM that they have created about daily life problem situation. This sub stage has come up as the result of other sub stages. If it has taught that MM could provide a sufficient solution for real life situation, it was decided that model was ideal for the solution of problem and solution was reported shortly. Students have mentioned the main points of modeling process approaches in the report.

When it was thought that MM couldn't provide a sufficient solution in validation it was turned back to the stage that was thought to be problematic. However, form the point of assumptions and main strategy solution was overviewed mostly. Solution process was continued until MM was created that provide an ideal solution.

Discussion

In the study which was carried out to explain exhibiting approaches of groups working with in technology-supported learning environment aimed at verification of model in modeling process and the thinking process that enable those; it was tried to give a comprehensive overview with five sub stages emerged in verification stage.

As Blomhøj and Jensen (2006), Blum and Niss (1991), Garofalo and Lester (1985) and Lesh and

Doerr (2003) stated "Verification of Model" was came out as the last stage of modeling process. In the study it was seen that students examine to what extent the real life results of the mathematical model that they have created meet with given problem situation with various approaches and in this way they investigate both results and their model. As Biccard and Wessels (2011), Mason (1988) and Berry and Houston (1995) stressed verification modeling is a stage that the process of examining model and output occur and it is generally ignored by students in modeling but should be considered. As Galbraith and Stillman (2006) stressed students always switch to other stages of modeling process in verification stage of our study. It was seen that the trueness was reviewed by comparing real life results obtained from mathematical models created in modeling process with groups' real life experience, real data in problem, animation, video, pictures given with problem and momentary measurement and predictions made by groups in process. This finding complies with Blomhøj and Jensen's (2006) result that in verification stage students generally benefit from experiences, observations, predictions or theoretical knowledge. In parallel with Borromeo-Ferri's (2006) study explaining that in modeling cycle verification can be done in two ways; intuitive and knowledge based; in this study it was seen that groups' intuitions and mathematical, technology and real life knowledge lies behind the approaches that groups showed in sub stages for verification. Intuitions, knowledge and skills of students had a great role in shaping sub stages by interacting each other. Results arising from the study, examining of unexpected situations sub stage in A/MM's real life results show parallelism with Galbraith and Stillman's (2006) "reconciliation unexpected situations with real situation" sub process of verification stage in modeling process. Beside Galbraith and Stillman's (2006) studies on grounded theory, this study is also thought to bring a different, rich and more comprehensive overview to the field. In this study the components that forming verification stage emerges as Real Life Solution, Real Life Problem Situation or Brief Solution Report. It was expressed that Computer software present images of mathematical model and help approaches to verification of model in later stages of modeling process (Lingerfjärd, 2000). In this study, in general video, picture and sections created from animations and in particular having a broad effect in solution; GeoGebra has important part in verification stage. Technology has provided a rich environment for students in explaining modeling process' real life situation and analyzing the behaviors and tendencies of the model they have created (Cheng, 2010).

Even if its' impact on five sub stages is not seen directly in our study as well; it is seen that technology has important contributions for the whole process. Baki (2002) also asserted that in modeling process use of video and animation can provide opportunities such as by redefining the conditions of groups monitoring results, comparison, model building, discovery and validation of new relations and features. In fact, computer software, as Lingefjärd (2000) pointed out, prevents the students to struggle with possible difficult operations. Modeling problems which have included real life data has caused to create models in a complex structure. However, students haven't waste any time on struggling with difficult operations by taking advantage of GeoGebra. This allows having richer conceptual and mental processes to occur in mathematical modeling. In addition, as Milli Eğitim Bakanlığı (MEB, 2006) pointed out, having students to take active part in mental processes such as guesswork, generalization, verification and a rich learning environment was created to show the accuracy of the results that they have reached in exploring environment.

In our opinion, GeoGebra software that working groups have used while solving the problems and verification stage of comprehensive modeling process which was created with video and pictures, can be tried to explain in a more comprehensive way by using GeoGebra 3D software and etc. and also by expanding problem variety in further studies. This study was carried out with prospective teachers instead having this study with high school students and mathematical teachers and also discussing how the verification stage would shape can provide important gain. The most suitable and effective learning environment design that enriches students' approaches in verification stage can be analyzed and this environment can be supported with technology.

References/Kaynakça

Abrams, J. P. (2001). Mathematical modeling: Teaching the open-ended application of mathematics. In A. A. Cuoco, & F. R. Curcio (Eds.), *The teaching mathematical modeling and the of representation*. *2001 Yearbook* (pp. 269-282). Reston, VA: NCTM.

Baki, A. (2002). Öğrenen ve öğretenler için bilgisayar destekli matematik**.** İstanbul: BİTAV-Ceren Yayın Dağıtım.

Berry, J., & Houston, K. (1995). *Mathematical modelling.* Bristol: J. W. Arrowsmith Ltd.

Biccard, P., & Wessels, D. C. J. (2011). Documenting the development of modelling competencies of grade 7 mathematics students. *International Perspectives on the Teaching and Learning of Mathematical Modelling, 1*(5), 375-383.

Blomhøj, M., & Jensen, T. H. (2006). What's all the fuss about competencies?. In W. Blum, P. L. Galbraith, H.-W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education.* (pp. 45-56). New York: Springer.

Blomh**ø**j, M., & Kjeldsen, T.H. (2006). Teaching mathematical modelling through project work - experiences from an in-service course for upper secondary teachers. *Zentralblatt für Didaktik der Mathematik, 38*(2), 163-177.

Blum, W. (2002). ICMI Study 14: Applications and modelling in mathematics education- discussion document. *Zentralblatt für Didaktik der Mathematik, 34*(5), 229-239.

Blum, W., & Ferri, R. B. (2009). Mathematical modelling: Can it be taught and learnt? *Journal of Mathematical Modelling and Application*, *1*(1), 45-58.

Blum, W., & Leiß, D. (2007). How do students and teachers deal with mathematical modelling problems? The example "sugarloaf". In C. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical Modelling (ICTMA 12): Education, Engineering and Economics* (pp. 222-231). Chichester: Horwood Publishing.

Blum, W., & Niss, M. (1991). Applied mathematical problem solving, modelling, application, and links to other subjects-state, trends and issues in mathematics instruction. *Educational Studies in Mathematics, 22*(1), 37-68.

Borromeo-Ferri, R. B. (2006). Theoretical and empirical differentiations of phases in the modelling process. *Zentralblatt für Didaktik der Mathematik-ZDM, 38*(2), 86-95.

Borromeo-Ferri, R. B. (2007). Personal experiences and extra-mathematical knowledge as an influence factor on modelling routes of pupils. *CERME 5 (2007) Working Group 1,* 2080-2089.

Charmaz, K. (2006). *Constructing grounded theory: A practical guide through qualitative analysis.* London: Sage.

Cheng, A. K. (2001). Teaching mathematical modelling in singapore schools. *The Mathematics Educator, 6*(1), 63-75.

Cheng, A. K. (2010). *Teaching and learning mathematical modelling with technology*. Nanyang Technological University. Retrieved from http://atcm.mathandtech.org/ep2010/invited/3052010_18134.pdf.

Clayton, M. (1999). Industrial applied mathematics is changing as technology advances. In C. Hoyles, C. Morgan, & G. Woodhouse (Eds.), *Rethinking the mathematics curriculum* (pp. 22-28). London: Falmer.

English, L. D. (2006). Mathematical modelling in the primary school: Children's construction of a consumer guide. *Educational Studies in Mathematics, 63*(3), 303-323.

Fox, J. (2006). A justification for mathematical modelling experiences in the preparatory classroom. P. Grootenboer, R. Zevenbergen, & M. Chinnappan (Eds.), *Proceedings 29th annual conference of the Mathematics Education Research Group of Australasia, 1*, 21-228.

Galbraith, P., & Stillman, G. (2006). A framework for identifying student blockages during transitions in the modelling process. *Zentralblatt für Didaktik der Mathematik-ZDM, 38*(2), 143-162.

Galbraith, P., Stillman, G., Brown, J., & Edwards, I. (2007). Facilitating middle secondary modelling competencies. In C. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical Modelling: ICTMA 12: Education, Engineering an Economics* (pp. 130-140). Chichester: Horwood Publishing.

Garofalo, J., & Lester, F. K. (1985). Metacognition, cognitive monitoring, and mathematical performance. *Journal for Research in Mathematics Education, 16*(3), 163-176.

Glaser, B. (1978). *Theoretical sensitivity.* Mill Valley, CA: Sociology Press.

Glaser, B., & Strauss A. (2006). *The discovery of grounded theory: Strategies for qualitative research.* New Brunswick, USA and London, U.K.: Aldine Transaction.

Hıdıroğlu, Ç. N. (2012). *Teknoloji destekli ortamda matematiksel modelleme problemlerinin çözüm süreçlerinin analiz edilmesi: Yaklaşım ve düşünme süreçleri üzerine bir açıklama*. Yüksek lisans tezi, Dokuz Eylül Üniversitesi, İzmir. http://tez2.yok.gov.tr/ adresinden edinilmiştir.

Kaiser, G., (2005, February). *Introduction to the working group "applications and modelling".* Paper presented at the Fourth Congress of European Research in Mathematics Education CERME4, St. Feliu de Guixols, Spain. Retrieved from http://www.mathematik.uni-dortmund.de/~erme/CERME4/CERME4_WG13.pdf.

Leiß, D., Schukajlow, R., Blum, W., Messner, R., & Pekrum, R. (2010). The role of the situation model in mathematical modelling-task analyses, student competencies and teacher interventions. *Journal für Mathematik Didaktik, 1*(1), 119-141.

Lesh, R., & Doerr, H. M. (Eds.). (2003). *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning and teaching.* Mahwah, NJ:Lawrence Erlbaum.

Lingefjärd, T. (2000). *Mathematical modeling by prospective teachers using technology* (Doctoral dissertation, University of Georgia). Retrieved from http://ma-serv.did.gu.se/matematik/thomas.htm.

Lingefjärd, T. (2006). Faces of mathematical modeling. *Zentralblatt für Didaktik der Mathematik-ZDM, 38*(2), 96-112.

Mason, J. (1988). Modelling: What do we really want pupils to learn? In D. Pimm (Ed.), *Mathematics, teachers and children* (pp. 201-215). London: Hodder & Stoughton.

Milli Eğitim Bakanlığı. (2006). *Ortaöğretim matematik dersi* öğretim *programı*. Ankara: Yazar.

National Council of Teachers of Mathematics. (1979). *Applications in school mathematics: 1979 yearbook.* Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (1998). *Principles and standards for school mathematics: Discussion draft.* Reston, VA: Author.

National Council of Teachers of Mathematics. (2000). *Curriculum and evaluation standards for school mathematics.* Reston, VA: NCTM Publications.

Niss, M. (1989). Aims and scope of applications and modelling in mathematics curricula. In W. Blum, J. S. Berry, R. Biehler, I. Huntley, G. Kaiser-Messmer, & L. Profke (Eds.), *Applications and modelling in learning and teaching mathematics* (pp. 22-31). Chichester: Ellis Horwood.

Peter-Koop, A. (2004). Fermi problems in primary mathematics classrooms: Pupils' interactive modelling processes. In I. Putt, R. Farragher, & M. McLean (Eds.), *Mathematics education for the Third Millenium: Towards 2010* (Proceedings of the 27th Annual Conference of the Mathematics Education Research Group of Australasia, pp. 454-461). Townsville, Queensland: MERGA.

Pugalee, D. K. (2004). A comparison of verbal and written descriptions of students' problem solving processes. *Educational Studies in Mathematics*, *55*(1), 27-47.

Punch, K. F. (2005). *Sosyal araştırmalara giriş nicel ve nitel yaklaşımlar* (Çev. D. Bayrak, H. B. Arslan, Z. Akyüz). Ankara: Siyasal Kitabevi.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. A. Grouws (Ed.). *Handbook of research on mathematics teaching and learning* (pp. 334- 370). New York: Macmillan.

Siller, H. S., & Greefrath, G. (2010). Mathematical modelling in class regarding to technology. *CERME 6 – Proceedings of the sixth Congress of the European Society for Research in Mathematics Education,* 108-117.

Skolverket (1997). *Kommentar till grundskolans kursplan och betygskriterier i matematik* [Commentary on the Comprehensive School Curriculum and Marking Criteria in Mathematics]. Stockholm: Liber Utbildningsförlaget.

Skovsmose, O. (1994). *Towards a philosophy of critical mathematics education.* Dordrecht, Netherlands: Kluwer.

Stillman, G. A., & Galbraith, P. L. (1998). Applying mathematics with real world connections: Metacognitive characteristics of secondary students. *Educational Studies in Mathematics*, *36*, 157-189.

Stillman, G., Galbraith, P., Brown, J., & Edwards, I. (2007). A framework for success in implementing mathematical modelling in the secondary classroom. *Mathematics: Essential Research, Essential Practice, 2*, 688-697.

Strauss, A., & Corbin, J. (1990). *Basics of qualitative research*. Thousand Oaks, CA: Sage.

Swedish Ministry of Education. (1994). *Kursplaner för grundskolan* [Syllabus for Subjects in the Comprehensive School Curriculum]. Stockholm, Fritzes.

Thomas, G. B., Weir, M. D., Hass, J., & Giordano, F. R. (2010). *Thomas calculus 1* (2. bs., Çev. R. Korkmaz). İstanbul: Beta Basım A.Ş.

Treffers, A. (1987). *Three dimensions. A model of goal and theory description in mathematics instruction- the wiskobas project.* Dordrecht, The Netherlands: Reidel Publishing Company.

Voskoglou, M. G. (2006). The use of mathematical modelling as a tool for learning mathematics. *Quaderni di Ricerca in Didattica, 16*, 53-60.

Williams, J. S. (1989). Real problem solving in mechanics: The role of practical work in teaching mathematical modelling. In M. Niss, W. Blum, & I. Huntley (Eds.), *Modelling applications and applied problem solving* (pp. 158-167). England: Halsted Press.

EK 1.

Matematiksel Modelleme Problemleri

Deneysel Modelleme

Boy-Ayak Uzunluğu Problemi

Kişi	Cinsiyet	Boy (cm)	Ayak Uzunluğu (cm)
1	K	160	25
2	E	111	15
3	K	160	23
4	K	152	23,5
5	K	146	24
6	K	157	24
7	E	136	21
8	K	143	23
9	E	147	20
10	E	133	20
11	K	153	25
12	E	148	23
13	E	125	20
14	K	150	20
15	E	183	28
16	E	184	25
17	E	125	18
18	K	140	20
19	E	170	27,5
20	K	168	25,5
21	E	131	23
22	E	149	23
23	K	156	21
24	K	130	19,5
25	K	142	22
26	K	159	24
27	K	145	25,5
28	K	162	25
29	E	149	22
31	E	126	20
32	E	150	24
33	E	170	26
34	K	141	21
35	K	123	20
36	K	122	19
37	E	125	20
38	K	133	20
39	E	165	25
40	K	131	20
41	K	134	17
42	E	158	25
43	K	170	25
44	K	125	15
45	K	135	21
46	K	138	19
47	E	134	20,5
48	E	145	22
49	K	171	25
50	K	181	24
51	K	139	19,5
52	E	147	25
53	E	134	19
54	K	164	24
55	E	127	19,5
56	K	138	23
57	E	180	24
58	E	159	26
59	K	151	23,5
60	E	165	29

Yukarıdaki tabloda 60 kişilik bir grubun cinsiyet, boy ve ayak uzunlukları verileri verilmiştir. Bu verilerden hareketle şu anda dünyanın en uzun boylu (247 cm) insanı yaklaşık olarak kaç numara ayakkabı giyer? Boyları aynı olan herhangi erkek ve kadının ayak uzunluklarının arasındaki ilişkiyi matematiksel olarak gösteriniz.

Teorik Modelleme

Salıncak Problemi

Salıncakta sallanan bir insanın sallanırkenki potansiyel enerjisindeki değişimi matematiksel olarak ifade ediniz. Videolardan da istediğiniz ölçüde faydalanarak tüm gerekçelerinizi ayrıntılı bir şekilde açıklayınız. (Öğretmen adaylarına problemle birlikte Ek 2'de kesitleri yer alan bir animasyon ve bir tane de video kaydı verilmiştir.)

Simülasyon Modelleme

Stat Problemi

Yakın zamanda ülkenizde düzenlenecek olimpiyat şampiyonası için yeni yapılacak stadın mimarlarından biri konumunda olduğunuzu düşünün. Sizden sahanın etrafını kaplayacak koşu pistini tasarlamanız isteniyor. Videodan ve fotoğraflardan istediğiniz ölçüde faydalanarak,

a) stadın koşu pisti (aynı anda 8 kişinin koşabileceği) olarak yapmayı düşündüğünüz modelinizi (şeklinizi) matematiksel modellerle destekleyerek oluşturunuz. (Koşu pisti oluşturma adına çizdiğiniz her şeklin matematiksel ifadelerle desteklenmesi gerektiğini unutmayınız.)

b) koşu pistini oluşturdunuz, şimdi de olimpiyatlarda bu statta 200 metre finalini koşacak 8 koşucunun koşu anını tasarlayınız. Adil bir yarış için koşunun nasıl yapılması gerekir? Koşucuların başlangıçtan bitişe konumları nasıl olmalıdır? Koşucuların yarış boyunca ki hareketlerini matematiksel olarak modelleyiniz. (Öğretmen adaylarına problemle birlikte Ek 3'de kesitleri yer alan bir animasyon ve 9 adet stadyum resmi verilmiştir.)

EK 2.

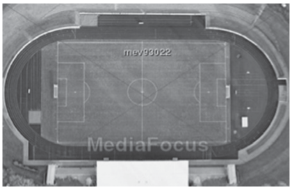
Salıncak Problemiyle Birlikte Verilen Videoların Kesitleri

Video-1'den Kesitler

Video-2'den Kesitler

EK 3.
Stat Problemiyle Birlikte Verilen Resimler ve Video Kesitlerinden Örnekler

Resimler



Video-1'den Kesitler